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ANALYSIS OF L SHAPED RC COLUMNS SUBJECTED TO BENDING WITH AXIAL LOAD USING MOMENT - CURVATURE RELATIONSHIP

SUBRAHMANYA. R. M[1], SHANMUKHA SHETTY[2], B. G. NARESH KUMAR[3]
VINDHYA NAYAK[4] & THUSHAR SHETTY[5]

[1]*Assistant Professor, Department of Civil Engineering, VCET, Puttur, India*

[2,5]*Assistant Professor, Department of Civil Engineering , NMAMIT, Nitte, Karkala, India*

[3]*Principal, Department of Civil Engineering, Maharaja Institute of Technology , Mysore, India*

[4]*Master Student, Department of Civil Engineering, SCEM, Mangalore, India*

ABSTRACT

This paper aims at analyzing the behavior of uni-axially and bi-axially eccentrically loaded columns of L shaped section coupled with axial loading using an analytical method in developing moment-curvature relationships. The conventional method of analyzing rectangular and circular columns subjected to bending using interaction diagrams is presented with Design aid for Reinforced concrete structures, SP-16. The applicability of this method with suitable modifications to L shaped columns is studied, this study provides an insight to the moment carrying capacity and curvature of L shaped columns, the moment-curvature relation was also compared with that of ETABS to estimate the possible deviations. The moment-curvature relations for columns with grade of concrete M20, M25 and M30 with grade of steel Fe 415 are presented, increase in grade of concrete, substantially reduces the ductility of column, though it results in an increased moment carrying capacity also results demonstrate that the moment carrying capacity of L shaped column is underestimated in ETABS.

***KEYWORDS:** Moment-Curvature Relationship; L Shaped Columns; Column Ductility & Load-Moment Interacton*

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Original Article

1. INTRODUCTION

In any structural system, columns are the significant structural elements and are to be designed with special care simply because their failure will result in the collapse of the structural system itself in contrary to the failure of beams which will be of localized nature. In this paper, a method of analyzing the complete behavior of columns of L shaped cross-section is proposed, wherein the analysis is based on the study of moment-curvature relationship derived from load-moment interaction diagrams developed using a strain based method. The sectional characteristics of special shaped columns like L columns are different in comparison to that of conventional rectangular and circular columns with their influence in load and moment carrying capacities mainly because the centre of gravity for L columns lies outside the section. Due to lack of codal provisions on behavioral and design aspects of L shaped columns in this work, emphasis is on establishing strength characteristics of L columns which is interpreted in the form of interaction diagrams and moment curvature relations. A computer analysis method was developed by Mon- Chen Liu (1), for the analysis of L shaped short RC columns and are validated using numerical and experimental results. A research on the behavior of beam column subjected to nonlinear axial force and biaxial bending considered for a probabilistic seismic evaluation the stress strain relationship for concrete is adopted from

Mander's model where the section is divided into a number of parallel fibers of concrete and steel (2). The effect of several types of cross-sections on the inter relationship of axial load, moment and curvature was studied and the results showed that there is a considerable influence of shape of cross section on the failure envelop (3). Research on columns of L shaped cross section subjected to combined biaxial bending and tension was carried out to evaluate ultimate strength of columns of L shape (4). An investigation on the flexural rigidity of RC columns with a proposed equation had its implication in the design of RC columns as it was suggested in the research work that the equation for EI stems out from the actual behavior of the column, which intern implies that the shape of the cross-section has its influence on behavior of column and its design (5). Inverse method of analysis was adopted to develop failure envelop which intern was used to evaluate the ultimate capacity of L shaped columns and the results were compared with the experimental investigation (6). The application of interaction diagrams and load contours developed for L shaped columns in designing L shaped columns was presented in a research work (7). It was presented in a research work that the actual buckling stress for a column can also be developed by exact theory which gives the failure envelop of steel columns of various shapes such as box, I, H and T shapes (8). The present work is to develop interaction diagrams and corresponding moment-curvature relations for L shaped RC columns and to study the influence of varying grades of concrete and steel on strength characteristics of these columns.

2. INTERACTION CURVE

The interaction diagram is a two dimensional graphical representation of external forces (bending and axial) that could result in the failure of a column. It represents the capacity of a column in carrying the maximum load and moment. The load-moment interaction curve is a representation of the failure envelop of an eccentrically loaded column of given properties. The co-ordinates of every point on the failure envelop corresponds to design strength values of ultimate load 'Pu' and ultimate moment "Mu" corresponding to a particular eccentricity "e" of loading. By analyzing the design interaction curve for a column section with given specifications, it is possible to arrive at a quick result on weather a column is safe or not for a given combination of factored load and factored moment. If a point with co-ordinates as factored load and factored moment fall within the design interaction curve, the column is considered to be safe. This implicates that, the design interaction curve represents a failure envelop, though, it must be noted that the term 'safe' actually implicates that the risk of failure estimated by the code IS 456-2000 (9) is acceptably low.

(A). Salient Points on the Column Interaction Curve

The salient points represented on the interaction diagram in figure 1 corresponds to the failure envelop.

- The point 1 in figure 1 corresponds to concentric loading condition with e=0 which is a case of pure axial compression, ultimate resisting moment M_{uR} = 0 and ultimate resisting load P_{uR} is indicated as P_{u0} given by the equation

$$P_{u0} = 0.446\, f_{ck}\, A_c + N\, f_y\, A_s \quad (1)$$

Where, A_c = Area of concrete in the section

A_s = Area of steel in the section as reinforcing bars

f_y and f_{ck}= Grade of steel and Grade of concrete respectively

N = Factor equal to 0.77 and 0.75 for steel grades of Fe 415 and Fe 500 respectively

- The point 1’ in figure 1 represents the loading condition with minimum eccentricity e_{min}.
- The point 3 in figure 1 represents the condition where the position of neutral axis from the highly compressed edge x_u is equal to the depth of the section i.e x_u = D, let the corresponding eccentricity e = e_d. For e < e_d, the entire section of the column is under compression.
- When the neutral axis is situated outside the section i.e xu > D, with 0.002< ε_{cu}< 0.0035.
- Where, ε_{cu} = ultimate strain in concrete under compression.
- For e > e_d, the neutral axis is situated within the section (x_u< D), with ε_{cu} = 0.0035 at the highly compressed edge.
- Point 2 in figure 1 represents general case, with neutral axis situated outside the section (e < e_d).
- The point 4 in figure 1 represents the balanced failure condition where both concrete and steel simultaneously reaching maximum yield representing pure flexure condition.
- The point 5 in figure 1 represents the pure bending condition and at this stage the concrete fails by pure tension.

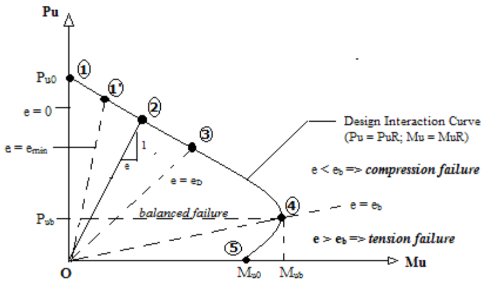

Figure 1: Idealized Moment-Curvature Relation

(B). Construction of a Design Interaction Curve

The co-ordinates of the design interaction curve, M_{uR} (on x axis) and P_{uR} (on y axis) can be calculated for any arbitrary position of neutral axis x_u. Having located (approximately) xu/D the co-ordinates of the design, interaction curve can be obtained with the following steps.

For the case of neutral axis lying inside the section K= Xu/D <1

Step1: Assume f_{ck}, f_y and diameter of reinforcing bars.

Step2: Axial load carrying capacity of concrete = 0.361*fck*b*d (2)

Step3: Axial load carrying capacity of steel

- ε_{sc} = 0.0035 [D-d’]/D (3)
- If ε_{sc} > 0.002 then the stress in concrete f_{cc}= 0.446f_{ck} and if ε_{sc} < 0.002 then f_{cc}= 446 ε_{sc} fck [1-250 ε_{sc}] (4)
- Total Axial load, Pu=(0.361f_{ck}BD)+∑Ast(stress in steel - stress in concrete) (5)
- Ultimate moment, Mu=0.361*f_{ck}*bd *[C.G-d’], where C.G is the distance of centre of gravity from highly compressed edge, b is the width and d is the depth of section above neutral axis.
- Curvature, Ø =(0.0035/X_u) (6)

- For the case of neutral axis situated outside the section $K= X_U/D >1$
- $(0.0035-0.75\varepsilon_{min})/(D+0.1D) = \varepsilon_{min} /0.1D$ (7)
- $\varepsilon_{cu} =0.0035-0.75\ \varepsilon_{min}$ (8)
- $C_c' =C_c/(f_{ck}BD)$ and $Y_c' =Y_c/D$, where B is the width of the section and D is the overall depth of the section
- Axial load carrying capacity of concrete=Cc'*fck*B*D (9)

Stress Calculation

- $\varepsilon_{sc} = \varepsilon_{cu}\ (D-d')/D$
- If $\varepsilon_{sc} > 0.002$ then the stress in concrete $f_{cc}= 0.446f_{ck}$ and if $\varepsilon_{sc} < 0.002$ then $f_{cc}= 446\ \varepsilon_{sc}$ fck $[1-250\ \varepsilon_{sc}]$
- Total Axial load, Pu= (Cc' f_{ck}BD) +∑Ast (stress in steel - stress in concrete)
- Moment, Mu= $(C_c' f_{ck}BD)[CG-Y_c'd]$
- Curvature, $Ø = (\varepsilon_{cu} / X_u)$
- The stress in steel corresponding to a particular stain is obtained from table A of SP-16:1980 (10)

Table 1: Coefficients C_c' and Y_c'

$K= X_u/D$	$C_c'= C_c/f_{ck}BD$	$Y_c'= Y_c/f_{ck}BD$
1	0.361	0.416
1.05	0.374	0.432
1.1	0.384	0.443
1.2	0.399	0.458
1.3	0.409	0.468
1.4	0.417	0.475
1.5	0.422	0.480
2.0	0.435	0.491
2.5	0.44	0.495
3.0	0.442	0.497
4.0	0.444	0.499

The values in Table 1 are the parameters specified in SP 16: 1980 (10), and the assumptions used in the design are as per the specifications of IS: 456-2000 (9).

3. MOMENT CURVATURE RELATION

Moment curvature plot is a graphical representation of the variation of moment of resistance with respect to curvature for a given section. This curve is unique for a given value of compressive load on the column. Moment of resistance is the moment developed by stress resultants of compressive and tensile nature. Curvature is the angle in radians made by the cross section in the vertical direction based on the strain in extreme fibre. A moment curvature plot has its application in non-linear static and non-linear dynamic analyses. It also has its application in understanding the ductility of steel sections and reinforced members. In earthquake resistant design, rotation capacity becomes a significant factor. This diagram indicates that there will be a residual energy dissipation potential even if the plastic capacity is reached. The member may continue to deform without losing its capacity but cannot carry a further load, this behavior is because of the ductility of the section. In earthquake resistant design, the ductility of the section plays a vital role.

(A). Salient Features of Moment Curvature Curve

The structural analysis is generally done by the classical elastic theory, but the design is done by the limit state method, taking material non linearity into consideration as specified in the code of practice IS: 456:2000(9). This method is applicable for both determinate and indeterminate structures, even under ultimate or factored load conditions in the linear range of moment-curvature relationship. For under reinforced RC sections this is valid as long as the stress in steel rebar stays within the elastic zone. However, once rebar yields, the behavior enter non-linear phase, and the simplified conventional linear elastic structural analysis is no longer valid. Inelastic analysis is therefore required to evaluate bending moment beyond the yielding point using a simplified limit state method which uses an idealized moment-curvature relationship as depicted in figure 2.

When the reinforcement in the tension zone yields the ultimate moment of resistance is assumed to have reached its critical limit. Further increase in strain will result in an increase in curvature without any increase in the ultimate moment of resistance. The yield zone acts as a potential hinge which undergoes continued rotation with a constant ultimate moment of resistance. Formation of further plastic hinges take place, and finally a critical combination of plastic hinges will occur resulting in the failure of concrete. Therefore, moment-curvature relationship is a significant modeling parameter for all flexural members in nonlinear structural analysis.

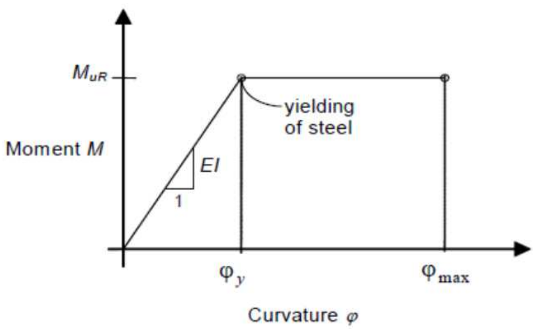

Figure 2: Idealized Moment-Curvature Relation

4. RESULTS

An example of L column is considered to study the effects of grade of concrete on the load and moment carrying capacity along with its effect on moment-curvature relationship, a sample result was also compared with that of ETABS result to analyze the possible deviation because of the whitney's stress block parameters adopted, in comparison to the parabolic rectangular stress block adopted in the proposed exact method.

(A). Example of L Column

- Grade of steel = 415 N/mm^2
- Dia of reinforcing bar = 16mm

(i). Highly Compressed Edge being 250mm Side of 1000mm Leg as Shown in Figure 3

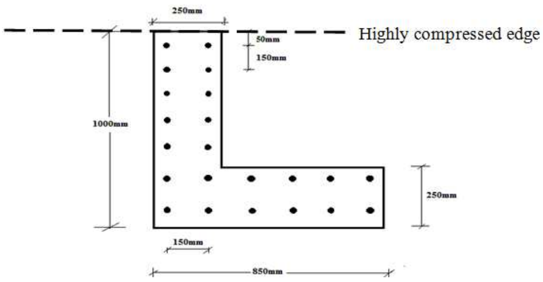

Figure 3: Column with Neutral Axis varied along 1000mm Side

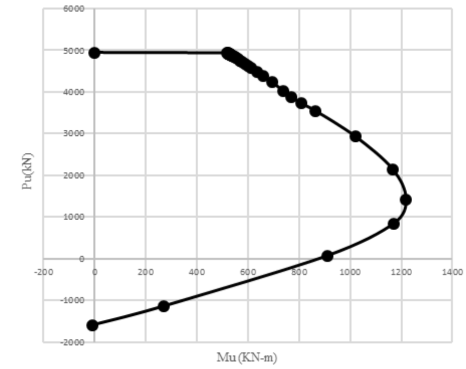

Figure 4: Interaction Diagram for M20 Grade

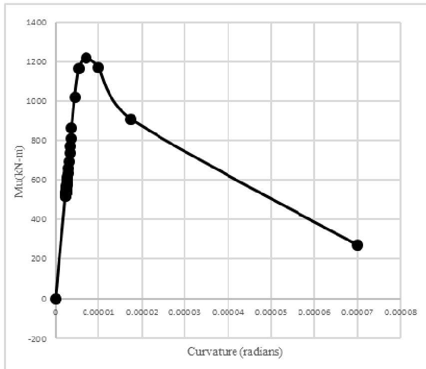

Figure 5: Moment Curvature Relation for M20 Grade

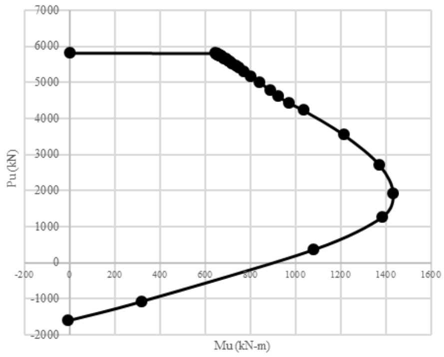

Figure 6: Interaction Diagram for M25 Grade

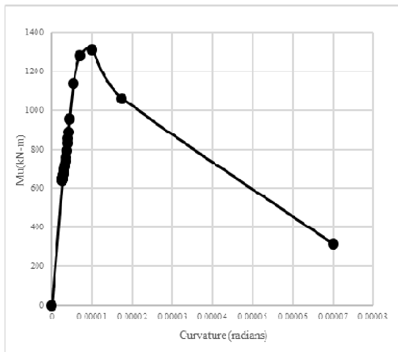

Figure 7: Moment Curvature Relation for M25 Grade

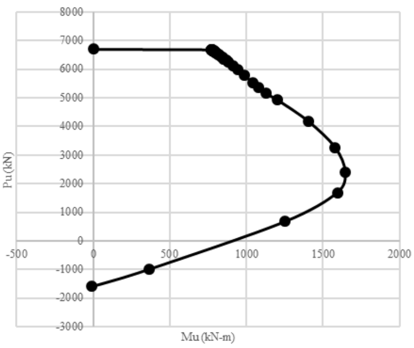

Figure 8: Interaction Diagram for M30 Grade

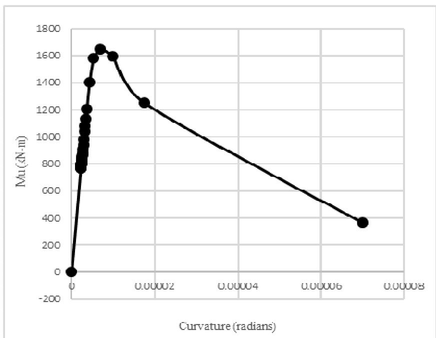

Figure 9: Moment Curvature Relation for M30 Grade

(ii). Highly Compressed Edge being 250mm Side of 850mm Leg as Shown in Figure 10

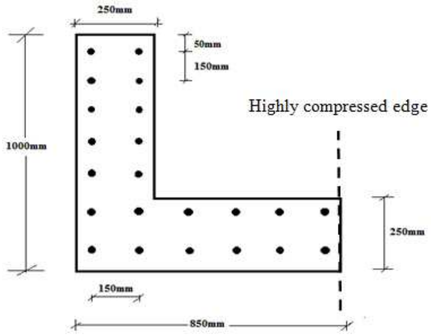

Figure 10: Column with Neutral Axis varied along 850mm Side

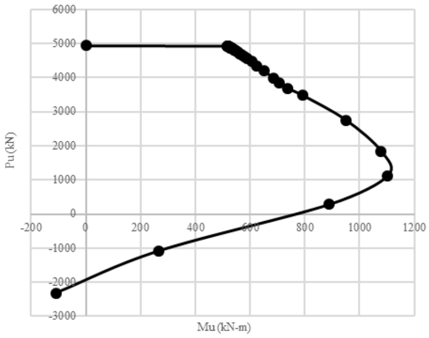

Figure 11: Interaction Diagram for M20 Grade

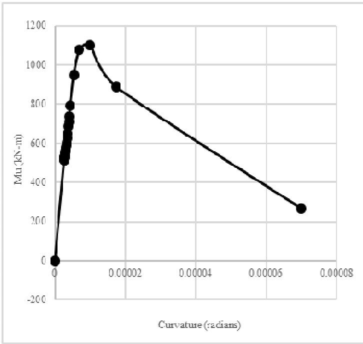

Figure 12: Moment Curvature Relation for M20 Grade

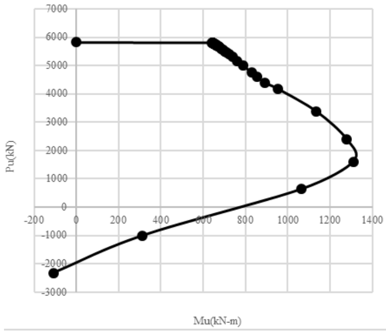

Figure 13: Interaction Diagram for M25 Grade

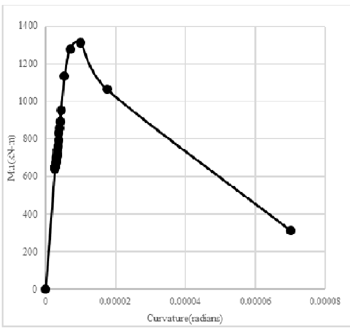

Figure 14: Moment Curvature Relation for M25 Grade

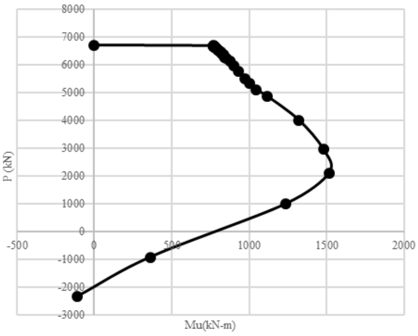

Figure 15: Interaction Diagram for M30 Grade

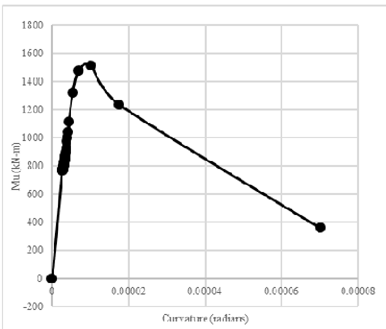

Figure 16: Moment Curvature Relation for M30 Grade

(iii). Highly Compressed Edge being 850mm Side of 850mm Leg as Shown in Figure 17

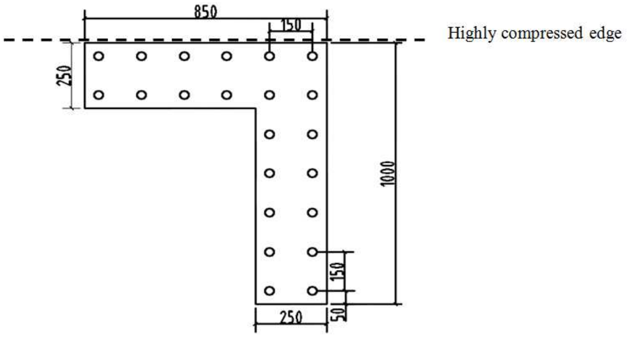

Figure 17: Column with Neutral Axis varied along 1000mm Side

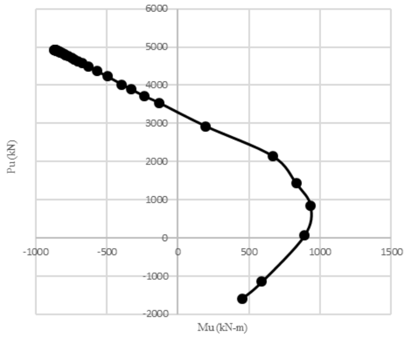

Figure 18: Interaction Diagram for M20 Grade

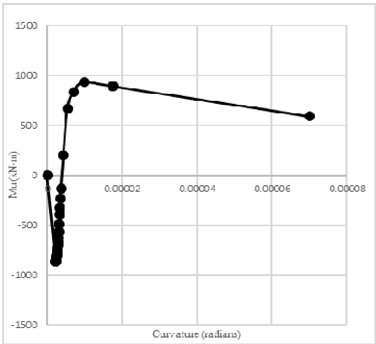

Figure 19: Moment Curvature Relation for M20 Grade

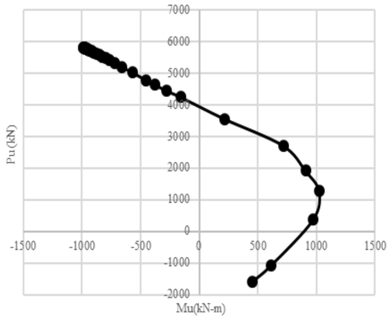

Figure 20: Interaction Diagram for M25 Grade

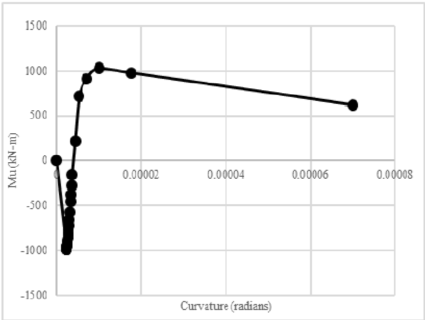

Figure 21: Moment Curvature Relation for M25 Grade

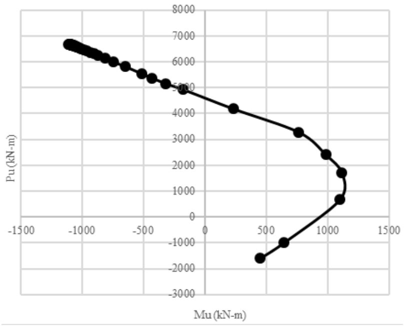

Figure 22: Interaction Diagram for M30 Grade

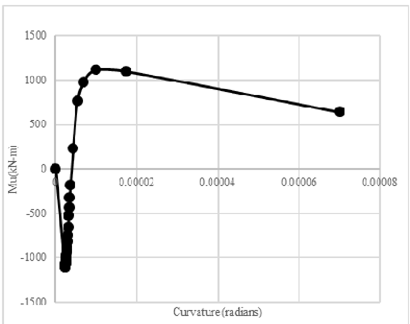

Figure 23: Moment Curvature Relation for M30 Grade

(iv). Highly Compressed Edge being 850mm Side of 1000mm Figure 24

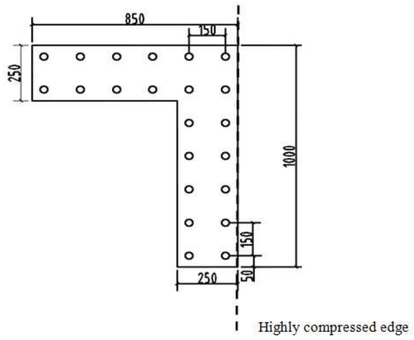

Figure 24: Column with Neutral Axis varied along 850mm Side

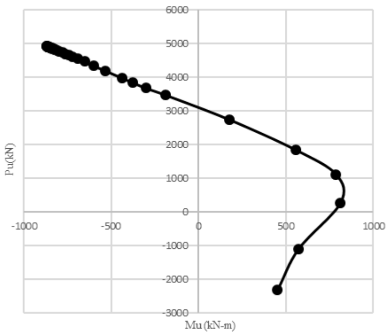

Figure 25: Interaction Diagram for M20 Grade

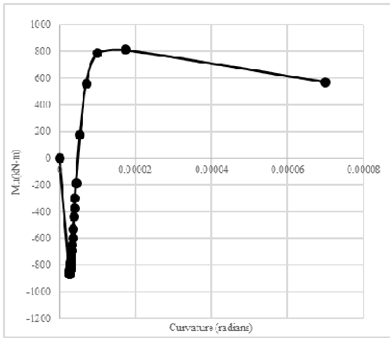

Figure 26: Moment Curvature Relation for M20 Grade

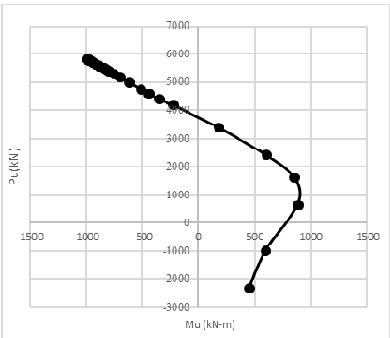

Figure 27: Interaction Diagram for M25 Grade

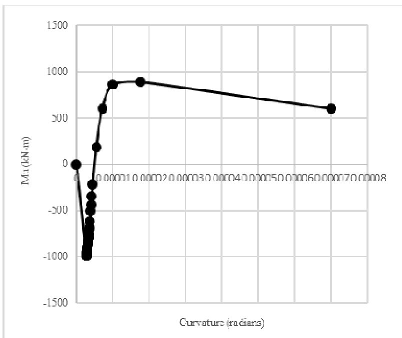

Figure 28: Moment Curvature Relation for M25 Grade

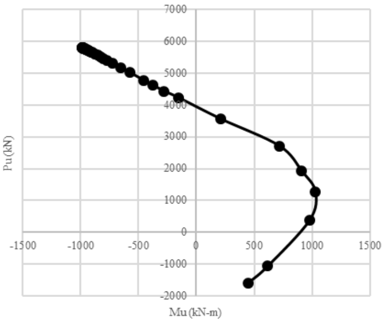

Figure 29: Interaction Diagram for M30 Grade

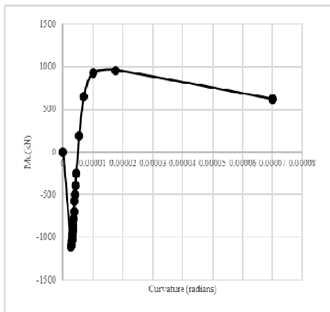

Figure 30: Moment Curvature Relation for M30 Grade

The interaction diagrams and moment curvature relationships developed for L column presented in the figure 4 to figure 30, for both the sides parallel to major and minor axis being highly compressed edge to address the column behavior corresponding to possible orientation and possible bending that could arise in practical case, and presents the influence of cross section shape and grade of concrete on load capacity, moment capacity and ductility of RC L shaped columns.

(B). Comparison of a Sample Result with Etabs-2015

Interaction diagram and the moment curvature relationship of the L column shown in Figure 6 and Figure 7 for M25 grade of concrete is compared with the results developed using ETABS-2015, to study the possible deviation of ETABS results from the exact method proposed in this paper.

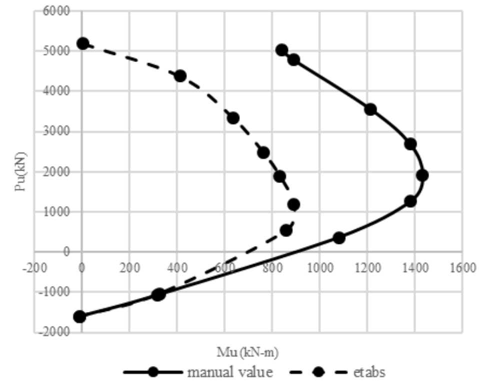

Figure 31: Interaction Diagram for M25 Grade

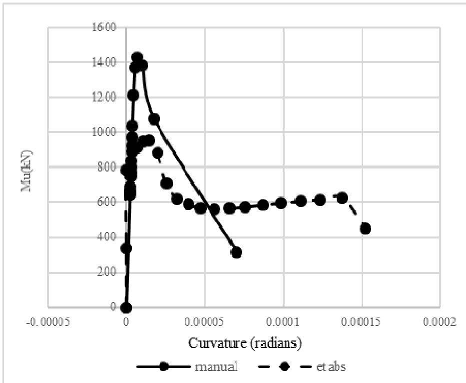

Figure 32: Moment Curvature Relation for M25 Grade

5. CONCLUSIONS

Interaction diagrams and moment curvature relations are developed for L shaped RC columns of unequal leg dimensions using the stress strain relationship adopted for classical layer decomposition method, the results will prove to be beneficial in understanding the behavior of L shaped columns and their ultimate load and moment carrying capacities, an interaction diagram developed for particular type of L shaped RC column with given specifications can be used to understand the range of load-moment combination where the column remains in compression failure region or in balanced failure region indicated by the first quadrant of interaction diagram corresponding to positive moment and positive load co-ordinates, it is advantageous to select the column with such specifications because the balanced region signifies that both concrete and steel would reach their ultimate values simultaneously and if the load-moment combination of the column falls in the compression region the corresponding failure will represent an under reinforced section in which the steel yields first developing cracks in the concrete which will give warning of failure of the column avoiding the risk of sudden collapse.

When highly compressed edge is considered as represented in figure 3 showed a higher moment carrying capacity in comparison to the case where highly compressed edge is considered as represented in figure 10, this behavior is mainly because of the longer leg dimension on which the highly compressed edge is considered as in figure 3, also the centre of gravity of the section is at a larger distance from extreme fibre which is considered to be the highly compressed edge in comparison to that for the orientation in figure 10, this would give a higher moment of resistance for the arrangement in figure 3, therefore it can be concluded that the orientation as in figure 3, with highly compressed edge as the web of longer leg shows a higher performance, this conclusion is arrived at by comparing results shown in figure 4 and figure 11.

For a particular value of curvature the moment carrying capacity is higher for the orientation as shown in figure 3, in comparison to that shown in figure 10, which could be established by comparing the results shown in figure 5 and figure 12.

Comparing the interaction diagram shown in figure 4, corresponding to the highly compressed edge as web in figure 3 with the interaction diagram in figure 18, corresponding to the highly compressed edge as the flange in figure 17, orientation as in figure 3 has a higher moment carrying capacity because the highly compressed edge is at a far distance from the centre of gravity of section in comparison to this distance corresponding to the orientation in figure 17. Also for the eccentricity corresponding to the position of neutral axis outside the section the orientation in figure 17 shows negative moment carrying capacities (moment in the opposite direction) whereas the moment carrying capacity is always positive for all the position of neutral axis for the orientation of columns shown in figure 3, these results could be observed from figure 4 and figure 18.

Increase in grade of concrete will increase the load and moment carrying capacity as indicated by the interaction diagrams, but the moment capacity for a given curvature reduces with increasing grade which represents that the ductility reduces with increased grade of concrete as represented by the moment curvature relationship.

The comparison of interaction diagram developed with that of ETABS shows that ETABS underestimates the moment carrying capacity of the column shown in figure 31, also for a particular curvature the moment capacity given by ETABS is on the lower side than the actual moment capacity for the same curvature as shown in figure 32.

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